

Problem 1.51

[Difficulty: 4]

1.51 The height of a building may be estimated by measuring the horizontal distance to a point on the ground and the angle from this point to the top of the building. Assuming these measurements are $L = 100 \pm 0.5$ ft and $\theta = 30 \pm 0.2^\circ$, estimate the height H of the building and the uncertainty in the estimate. For the same building height and measurement uncertainties, use *Excel's Solver* to determine the angle (and the corresponding distance from the building) at which measurements should be made to minimize the uncertainty in estimated height. Evaluate and plot the optimum measurement angle as a function of building height for $50 \leq H \leq 1000$ ft.

Given: Data on length and angle measurements

Find: Height; Angle for minimum uncertainty in height; Plot

Solution:

The data is: $L = 100$ -ft $\delta L = 0.5$ -ft $\theta = 30$ -deg $\delta\theta = 0.2$ -deg

Uncertainties: $u_L = \frac{\delta L}{L}$ $u_L = 0.5\%$ $u_\theta = \frac{\delta\theta}{\theta}$ $u_\theta = 0.667\%$

The height is: $H = L \cdot \tan(\theta)$ $H = 57.7$ -ft with uncertainty $u_H = \sqrt{\left(\frac{L}{H} \cdot \frac{\partial}{\partial L} H \cdot u_L\right)^2 + \left(\frac{\theta}{H} \cdot \frac{\partial}{\partial \theta} H \cdot u_\theta\right)^2}$

Hence with $\frac{\partial}{\partial L} H = \tan(\theta)$ $\frac{\partial}{\partial \theta} H = L \cdot (1 + \tan(\theta)^2)$ $u_H = \sqrt{\left(\frac{L}{H} \cdot \tan(\theta) \cdot u_L\right)^2 + \left[\frac{L \cdot \theta}{H} \cdot (1 + \tan(\theta)^2) \cdot u_\theta\right]^2}$

Evaluating $u_H = 0.949\%$ and $\delta H = u_H \cdot H$ $\delta H = 0.548$ -ft

The height is then $H = 57.7$ -ft ± 0.548 -ft

To plot u_H versus θ for a given H we need to replace L , u_L and u_θ with functions of θ . Doing this and simplifying

$$u_H(\theta) = \sqrt{\left(\tan(\theta) \cdot \frac{\delta L}{H}\right)^2 + \left[\frac{\delta\theta}{\tan(\theta)} \cdot (1 + \tan(\theta)^2)\right]^2}$$

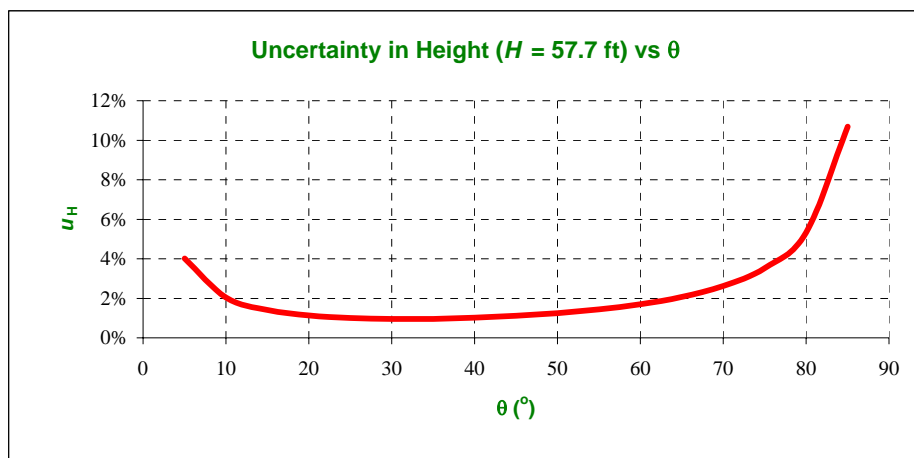
Given data:

$H = 57.7$ ft
 $\delta L = 0.5$ ft
 $\delta\theta = 0.2$ deg

For this building height, we are to vary θ (and therefore L) to minimize the uncertainty u_H .

Plotting u_H vs θ

θ (deg)	u_H
5	4.02%
10	2.05%
15	1.42%
20	1.13%
25	1.00%
30	0.95%
35	0.96%
40	1.02%
45	1.11%
50	1.25%
55	1.44%
60	1.70%
65	2.07%
70	2.62%
75	3.52%
80	5.32%
85	10.69%

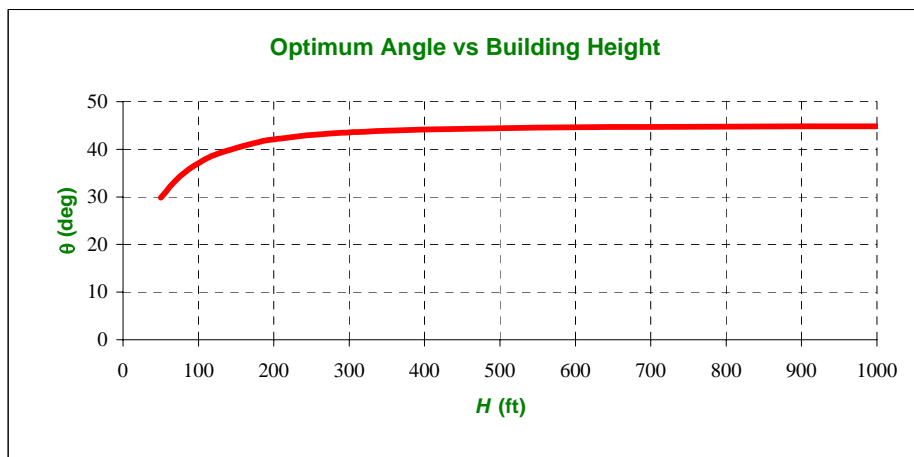


Optimizing using *Solver*

θ (deg)	u_H
31.4	0.947%

To find the optimum θ as a function of building height H we need a more complex *Solver*

H (ft)	θ (deg)	u_H
50	29.9	0.992%
75	34.3	0.877%
100	37.1	0.818%
125	39.0	0.784%
175	41.3	0.747%
200	42.0	0.737%
250	43.0	0.724%
300	43.5	0.717%
400	44.1	0.709%
500	44.4	0.705%
600	44.6	0.703%
700	44.7	0.702%
800	44.8	0.701%
900	44.8	0.700%
1000	44.9	0.700%



Use *Solver* to vary ALL θ 's to minimize the total u_H !

Total u_H 's: 11.3%